MOIRÉ PATTERNS

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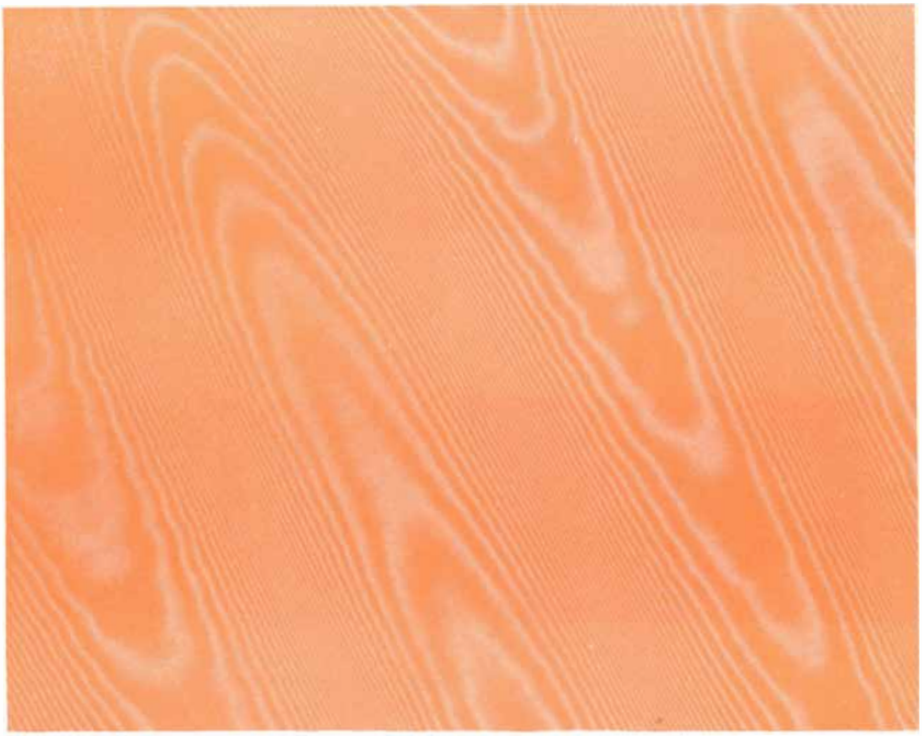
MOIRÉ PATTERNS

They are produced when figures with periodic rulings are made to overlap. A study of their basic properties reveals that they can illuminate many problems of scientific interest

by Gerald Oster and Yasunori Nishijima

When one looks through a window screen that happens to be in front of another window screen, one sees a curious pattern that results from a combination of the lines in the two screens. Such patterns are called moirés, and they are produced whenever two periodic structures are overlapped. *Moiré* is the French word for "watered"; in English it is most frequently heard in the term "moiré silk," a fabric that has a shimmering appearance resembling the reflections on the surface of a pool of water. Authentic moiré silk (*moiré antique*) is produced from a glossy fabric with a pronounced weave of parallel cords. The fabric is folded so that the cords are nearly aligned and the two layers are pressed so as to engrave the parallel weave of one onto that of the other. When the material is unfolded, it displays a moiré pattern due to the superposition of slightly misaligned parallel lines.

Moiré patterns are nowhere more familiar in daily life than they are in Japan. They appear not only in moiré silks (which were made in the Orient long before they were known in France) but also in two-ply summer kimonos, the pattern of which shimmers with the movements of the wearer, in woven baskets and in the overlapping layers of half-raised bamboo blinds. Such patterns have long fascinated the authors of this article, and we recently undertook to investigate their fundamental properties. As we looked into the matter we soon realized that moiré patterns could be exploited for a number of practical purposes in the laboratory and elsewhere. It is our impression that a systematic exploration of the moiré phenomenon and its potential uses would be highly rewarding.

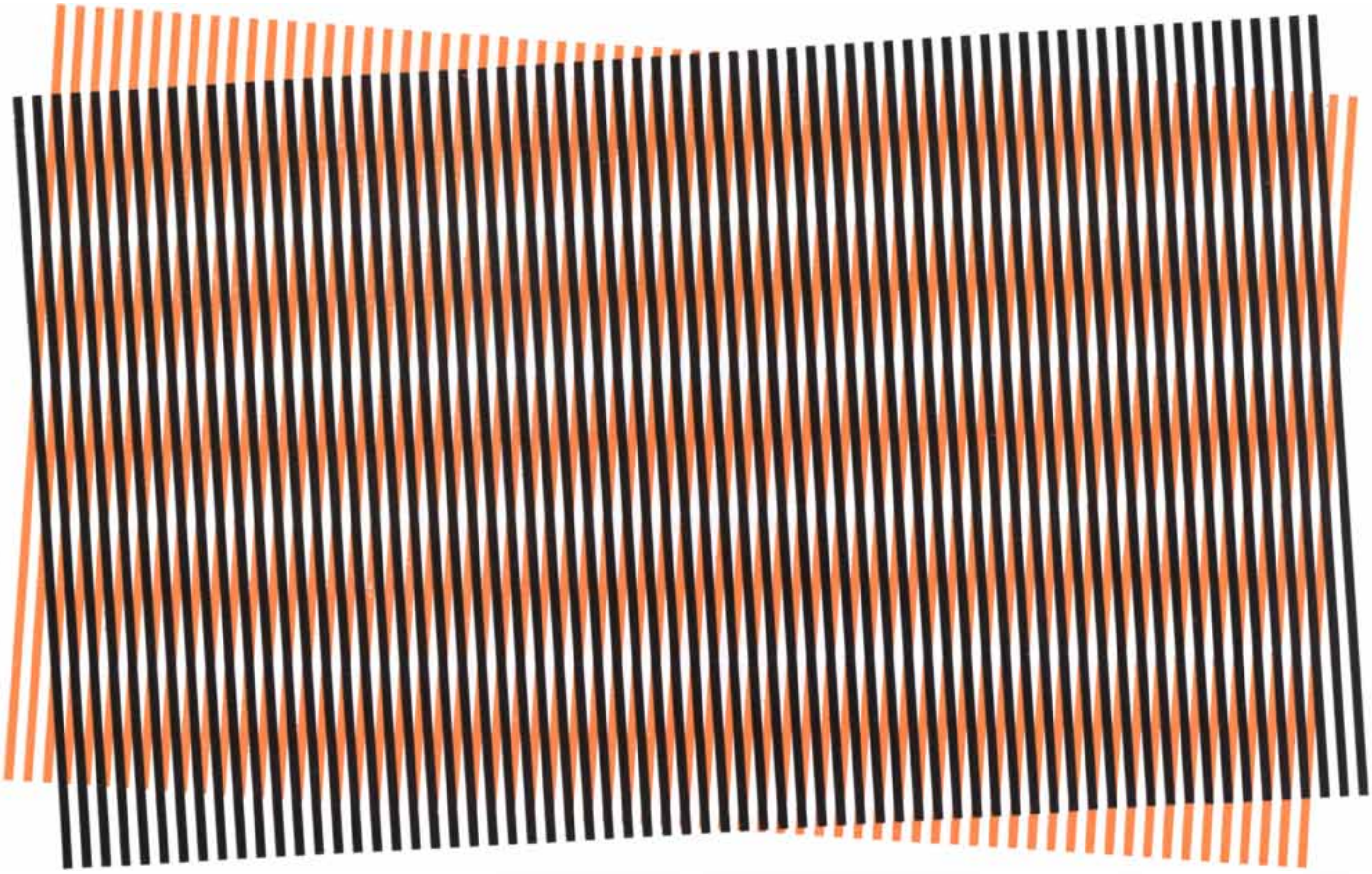


MOIRÉ ANTIQUE **is the authentic form of moiré, or "watered," silk in which the pattern is formed by doubling over a glossy corded fabric and pressing the facing surfaces together. When the fabric is unfolded, the superpositioning of two ribbed patterns creates the moiré.**

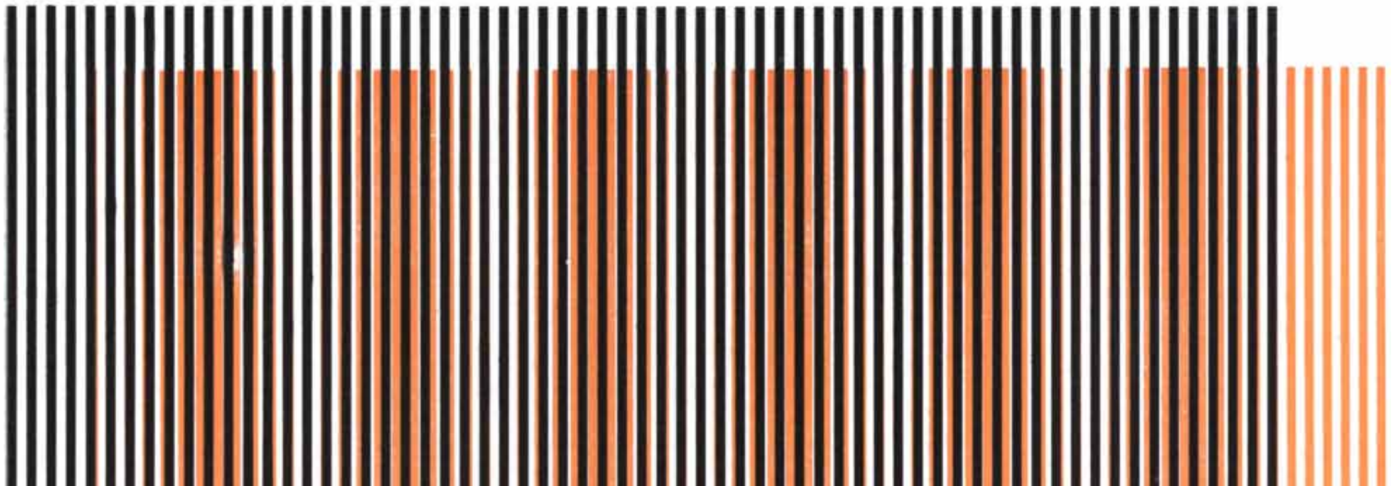
Most moiré patterns are generated by figures that consist of lines, but lines are not strictly necessary. The only general requirement for a moiré pattern is that the interacting figures have some sort of solid and open regions. The solid regions can be lines (straight, curved or wiggly), dots or any other geometric form. Most of this discussion, however, will be limited to moiré patterns resulting from lines, either straight or curved.

In the typical moiré pattern the moiré effect materializes when two sets of straight lines are superposed so that they intersect at a small angle [*see top illustration on opposite page*]. If the superposed lines are nearly parallel, a tiny displacement of one of the figures will give rise to a large displacement in the elements of the moiré pattern. In other words, the displacement is magnified. This phenomenon has far-reaching implications in many disciplines of science. For example, we have developed a system of lensless optics in which the bending of light by the object under examination causes a large change in the resulting moiré pattern.

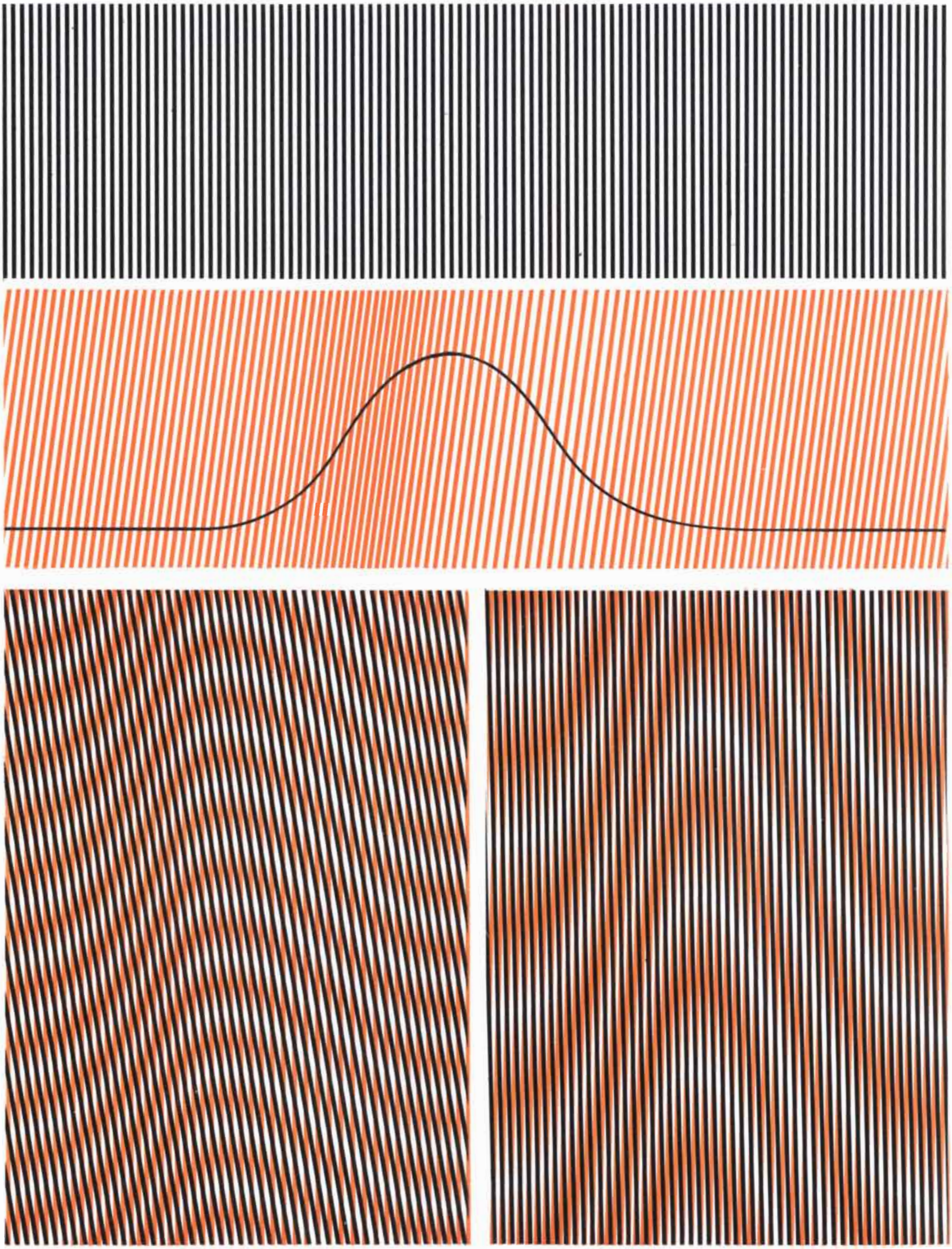
A moiré pattern can be regarded as the mathematical solution to the interference of two periodic functions; hence the moiré technique can be used as an analogue computer. When line figures representing periodic functions are moved about in a continuous manner,



SIMPLE MOIRÉ PATTERN appears when one set of equally spaced straight lines is made to intersect at a small angle with an identical set. Small changes in the angle of intersection of the two figures produce large changes in the spacing of the moiré fringes.



MOIRÉ COMPOSED OF BEATS is produced from nonintersecting parallel lines when the spacing of one set differs from that of another. Because moiré beats magnify small displacements they quickly reveal whether or not two sets of rulings are identical.



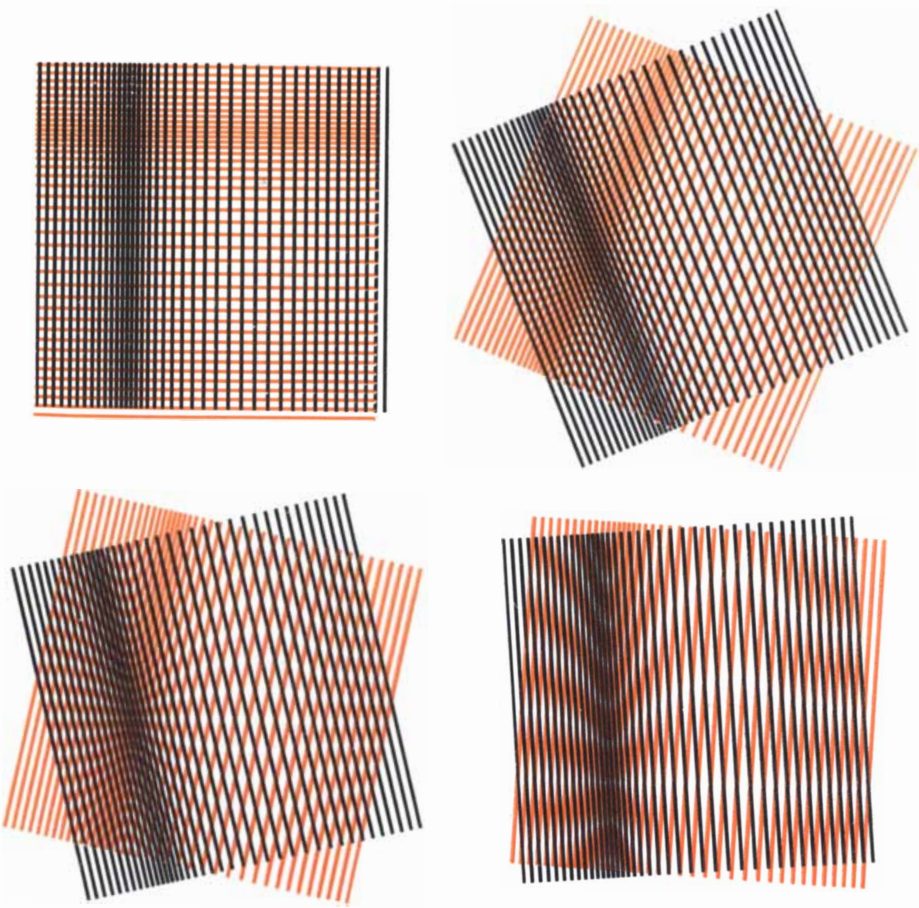
GAUSSIAN CURVES can be created by overlapping one set of lines of equal spacing (*top*) with a second set of lines whose spacings are derived from a Gaussian curve (*second from top*). The second figure is made by drawing a set of equally spaced vertical lines (*not shown*) through a Gaussian curve and then drawing slanting parallel lines through the points where the vertical lines intersect the curve. When the resulting set of rulings is placed over the regular rulings, a series of Gaussian curves is reproduced in a moiré pattern (*lower left*). Reducing the angle of intersection between the two figures steepens the curvature (*lower right*).

the resulting moiré patterns provide a continuous series of curves corresponding to the solutions of mathematical problems [*see illustrations on cover and on opposite page*]. If more than two figures are used, one can obtain with no effort a moiré pattern containing the solution to a multifunctional problem. The use of moiré techniques in this way would be applicable to complex problems involving electromagnetic radiation, acoustical waves or water waves.

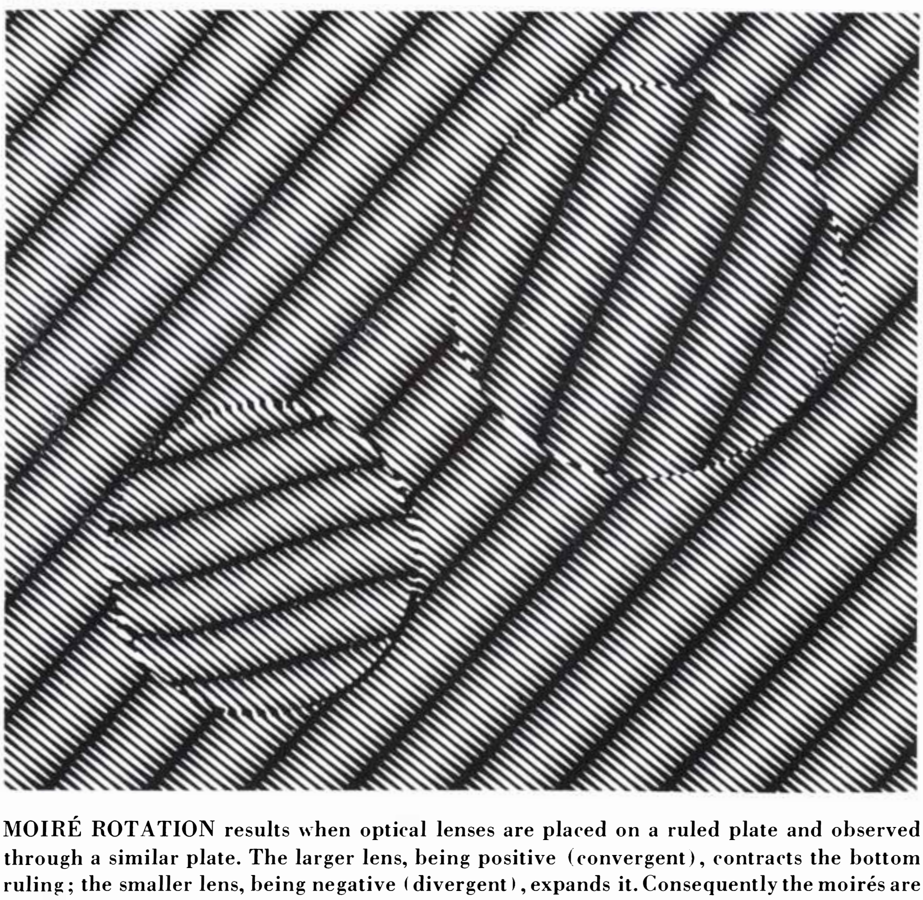
The simplest form of moiré pattern arises from the parallel superposition of two sets of equidistant parallel lines. This kind of pattern is represented by the moving bands one may see when one drives over a bridge at the side of which are two parallel railings consisting of vertical bars. To the observer the spaces between the bars of the nearer railing appear to be somewhat larger than those between the bars of the farther one. Whenever one bar in the nearer railing catches up with and fills a space in the other railing, a beat is observed. This is demonstrated in the bottom illustration on page 55, in which a beat occurs when a line of one figure falls exactly between two lines of the other figure. When the lines are not wide enough to fill a space completely, a beat is produced by an apparent broadening of the lines as the two figures move out of phase. It is obvious that the more closely the two sets of rulings match each other, the farther apart the beats are. Thus if the rulings are a millimeter apart but one set is in error in every spacing by one micron (.001 millimeter), the beat will occur every meter. Hence the moiré pattern represents an enormous magnification (in this case a million times) of the difference in length of the spacings. This system provides an extremely sensitive means of visually detecting minute differences in almost identical repeating figures.

As long ago as 1874 the British physicist Lord Rayleigh suggested that moiré patterns could be used to test the perfection of ruled diffraction gratings. In recent years the technique has been extensively employed, notably at the National Physical Laboratory in England, to test the fidelity of the replica technique for making inexpensive diffraction gratings for optical monochromators. By placing the plastic replica over the original grating one can immediately see any periodic errors made by the ruling engine or any distortions resulting from the production of the replica.

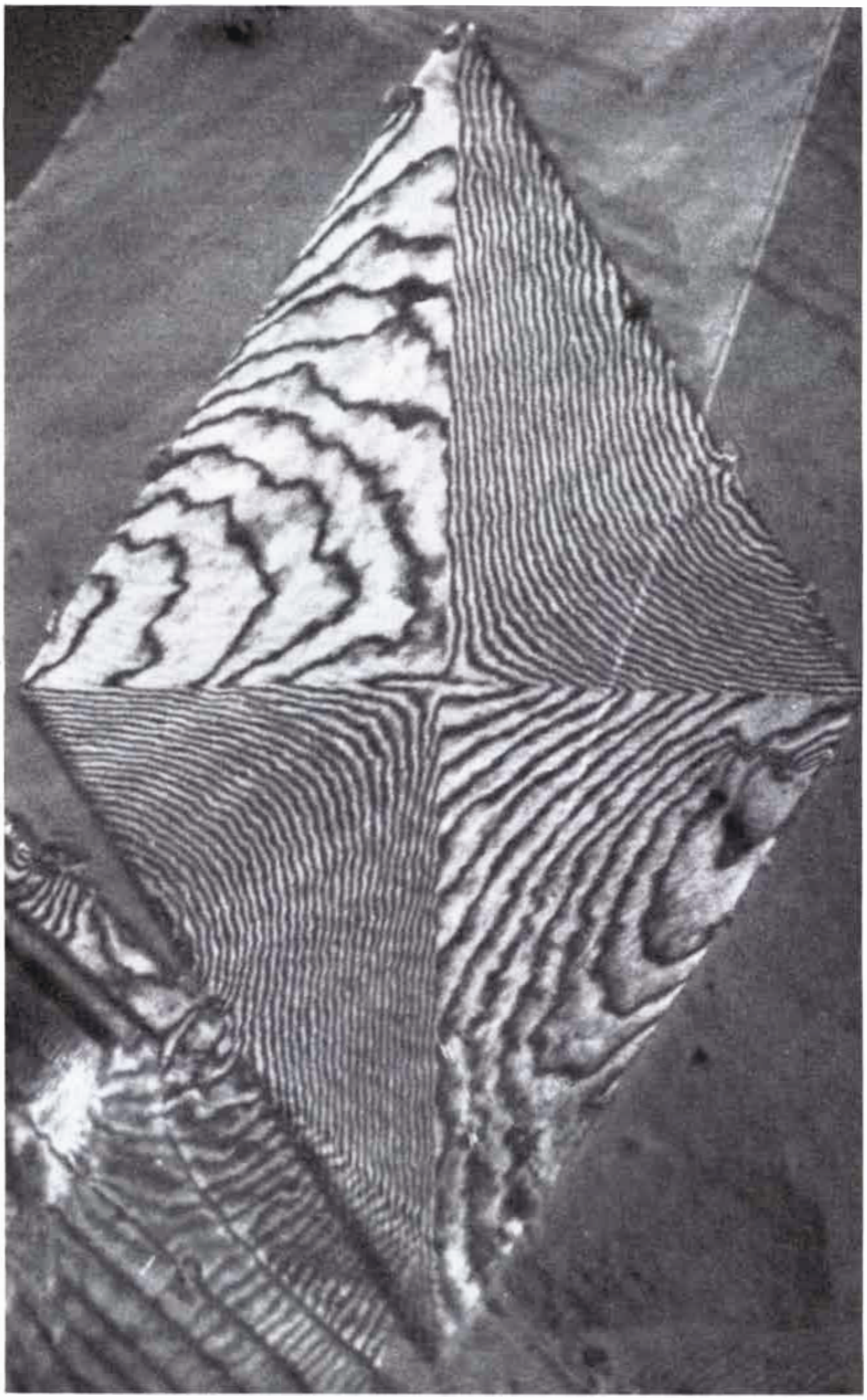
An interesting effect can be produced by taking two rulings that differ slight-



MOIRÉ ON COVER was made by overlapping two identical sets of ruled figures derived from a Gaussian curve, similar to the figure second from the top on the opposite page. This sequence shows how the complex pattern on the cover emerges when the two figures are rotated from a position 90 degrees out of phase (*top left*) until they are almost aligned.



MOIRÉ ROTATION results when optical lenses are placed on a ruled plate and observed through a similar plate. The larger lens, being positive (convergent), contracts the bottom ruling; the smaller lens, being negative (divergent), expands it. Consequently the moirés are rotated in opposite directions. A wavy moiré indicates that the lens contains aberrations.



MOIRÉ CREATED BY TWO CRYSTALS appears in this remarkable electron micrograph made by V. F. Holland of the Chemstrand Research Center, Inc., in Durham, N.C. The picture shows a crystal of polyethylene grown on a layer crystal of the same polymer. The moiré appears because the atomic lattices of the two crystals happen to be in almost direct alignment; the closer the alignment, the wider the spacing between the moiré fringes.

ly in line spacing and moving them with respect to each other. What one sees is a moiré beat that moves much faster than the rulings themselves. The beat is analogous to the beat produced by two waves of slightly different wavelength moving in the same direction. It is evident that the beat can move faster than the individual waves. For example, the beat produced by a collection of light waves forms a wave packet whose "group velocity" exceeds that of light itself. In quantum mechanics such wave packets play an important role in the theory of atomic structure.

Recently moiré patterns produced by stationary beats have been observed in electron micrographs of crystals. To obtain these patterns one must place a thin crystal atop another of the same material; if the crystals are of substantial thickness, the beam energy of the instrument must be several times higher than the voltage of conventional electron microscopes. The extra voltage is needed to penetrate the crystals, and the moiré patterns appear only when the lattices of the two crystals happen to be nearly superposed. The patterns represent the interference occurring when the electrons pass through two almost perfectly matched lattices formed by the atoms in the two crystals. These patterns supply far more detail than can be seen in ordinary electron micrographs [*see illustration at left*].

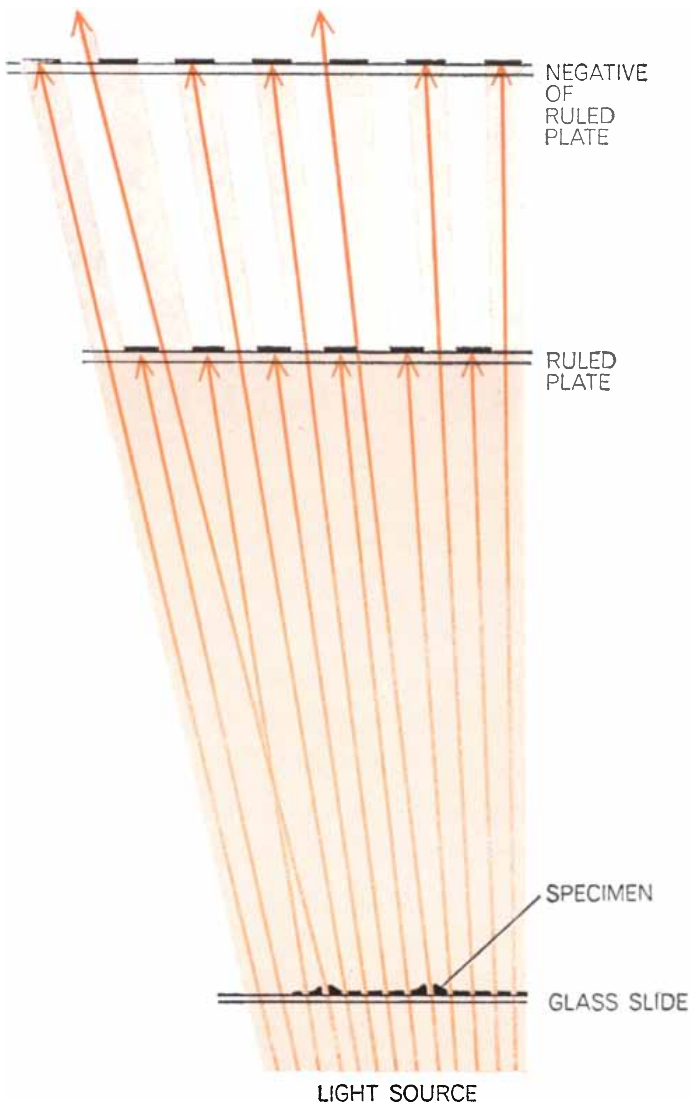
Since the moiré pattern arises from the repeating structure of the crystal, any dislocation that disturbs this regularity will be manifested in the pattern. The magnification implicit in the moiré pattern makes it possible to see dislocations that amount to less than the diameter of an individual atom, or less than one angstrom unit (.01 millionth of a centimeter). This is a factor of 100 or 1,000 better than the resolving power of the electron microscope itself.

Another aid to crystallography may result from using moiré patterns to solve complex equations. In X-ray diffraction studies X rays are deflected by planes of atoms in a crystal, and the deflected wavelets are recorded photographically. Where wavelets reinforce one another the plate is blackened, and where they cancel one another the plate remains clear. The crystallographer's problem is to deduce from such black and white patterns the location of atoms in the crystal. For any given image there are many possible solutions because the phase relations of the waves forming the image are unknown. This simply means that one does not know, in the case of a given wave, whether the plate is recording the crest of the wave or the trough. It is easy to show mathematically that trying out different phase relations, in seeking a solution to the crystal structure, is equivalent to shifting the relation of two periodic figures to form a moiré pattern. In this case the patterns are those formed when the rulings cross at a small angle. The great value of using moiré patterns for this purpose is that one can continuously vary the specifications of the moiré system, thereby obtaining a near infinity of solutions. The correct solution to a crystal-structure problem is one that satisfies all the known restrictions on the way atoms may be fitted together.

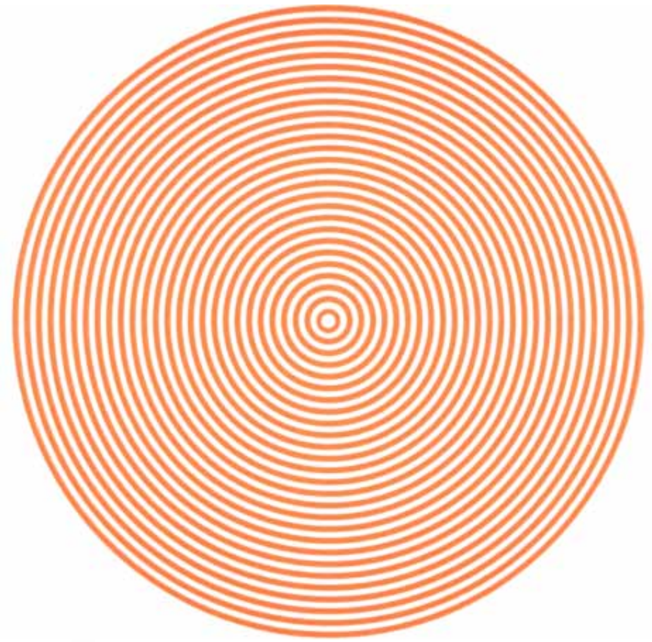
Moiré patterns can be used to demonstrate and measure an interesting property of crystals. All crystals except cubic ones exhibit more than one refractive index. For example, a rhomb of calcite has two refractive indexes, one corresponding to the "ordinary" ray (which passes straight through the crystal) and the other to the "extraordinary" ray (which is bent in passage). Each ray bends different wavelengths of light by different amounts. If a rhomb of calcite is placed between two sets of parallel rulings and illuminated with white light, the extraordinary ray will generally produce a colored moiré pattern. (Although the color is easily seen, it is difficult to show photographically.) The color will not appear, however, when the lines of the ruling nearest the observer lie along the line connecting the points of emergence of the two rays.

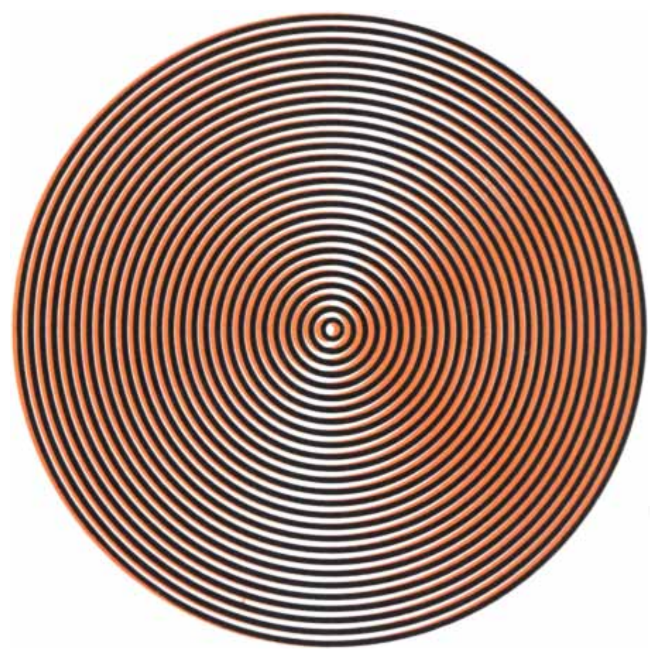
If the rhomb of calcite is placed on a single plate that is inscribed with concentric circles instead of straight lines, the overlapping figures produced by the two rays give rise to moiré patterns consisting of hyperbolas. The hyperbolas result from the intersection of overlapping circles and therefore depend on the center-to-center distances of the figures. Thus by counting the number of hyperbolas one can determine the relative displacements caused by the ordinary and extraordinary rays and hence the difference in refractive index of the two rays. This difference is called the birefringence of the crystal.

In another class of applications moiré patterns provide immediate visible evidence of transient changes in a medium placed between two sets of rulings. The technique is much simpler than those customarily employed, which require complex optical systems of either the interference or the schlieren type. Suppose, for example, that one would like to follow the rate at which some compound, say sugar, dissolves in a liquid medium. A flat-sided vessel of the liquid is placed between two sets of rulings, which are adjusted to produce a moiré pattern. A piece of sugar is then suspended in the liquid. As it dissolves it changes the refractive index of the liquid in its immediate vicinity and the changed index causes a proportional bending of the light rays passing through the vessel. The bending in turn distorts the moiré pattern [*see illustration on page 63*]. The distortion in the pattern is related in a direct and simple way to the change in refractive index. The moiré technique could readily be applied to many laboratory procedures,



LENSLESS MOIRÉ MICROSCOPE has been devised by the authors. It employs two precisely aligned ruled plates, one the negative of the other, that totally block light from a point source. If a transparent specimen is placed in the light beam, it bends the rays according to local variations in its refractive index, thereby making some of the light spill around the ruling of the top plate (*left*). Viewing the top plate, one sees a bright moiré on a dark background (*right*). The moiré is a refractive-index map of the specimen.



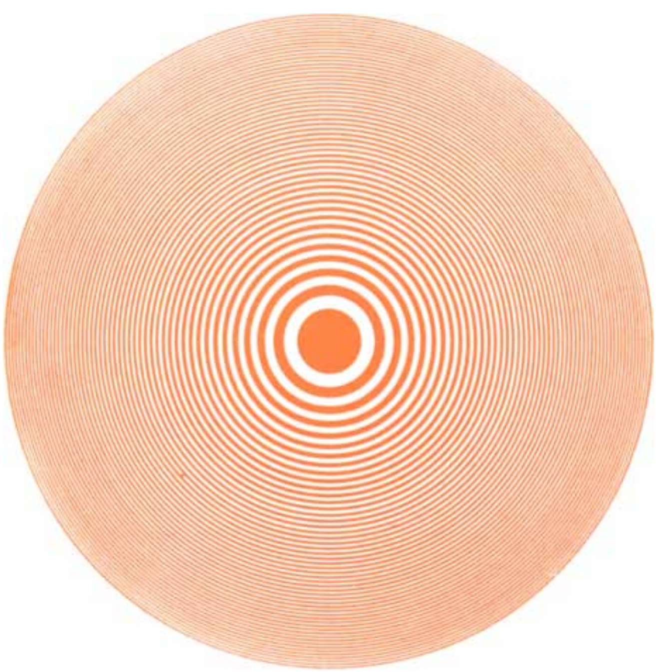


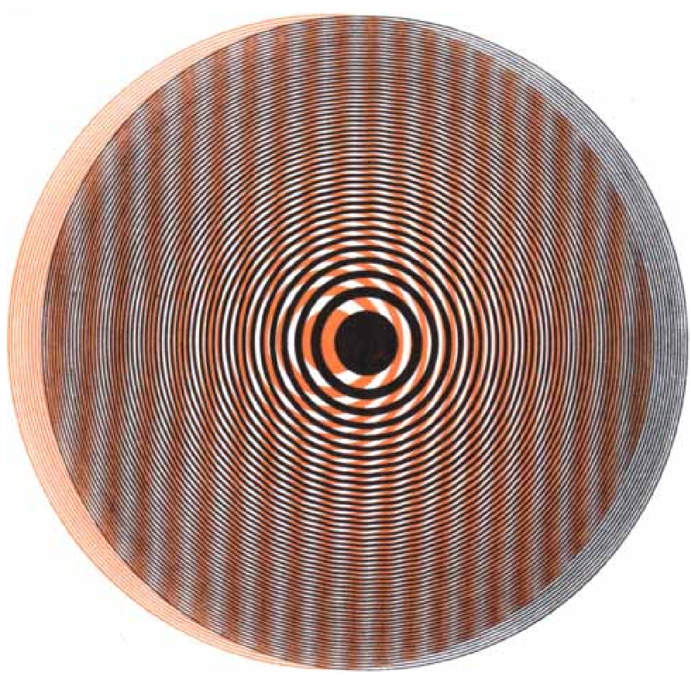
MOIRÉS BASED ON CIRCLES can provide models of many physical phenomena, such as the figures created when two waves are generated in phase from different centers. The set of circles at the left provides the basis for the three patterns shown in subsequent

such as ultracentrifugation, diffusion and electrophoresis, that require a continuous monitoring of changes in refractive-index gradients. The size and shape of proteins, nucleic acids and other giant molecules are commonly calculated by combining diffusion and sedimentation rates, which are derived by plotting such gradients.

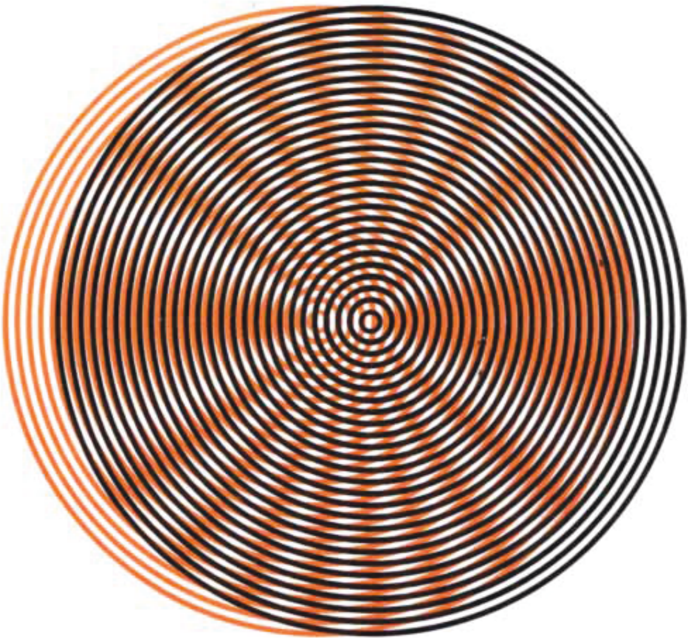
A related use of moiré patterns would be in the examination of biological specimens that have refractive indexes close to that of water. By the use of a suitable apparatus the refractive-index gradient of the specimen can be made to show up as a bright moiré pattern on a black background. An apparatus in which the specimen is illuminated by a divergent beam of light from a point source would constitute a lensless microscope [*see illustration on preceding page*].

A further use of the same principle provides a quick means for testing lenses. The lens to be examined is placed between two sets of rulings. A positive (convergent) lens magnifies the moiré pattern; a negative (divergent) lens reduces it. Positive and negative lenses rotate the moiré in opposite directions, and the angle over which the pattern is turned is proportional to the focal length of the lens. If the lens contains distortions, it will bend the lines of the moiré pattern. If it contains chromatic aberrations, the moiré pattern will exhibit color fringes. With little more difficulty one

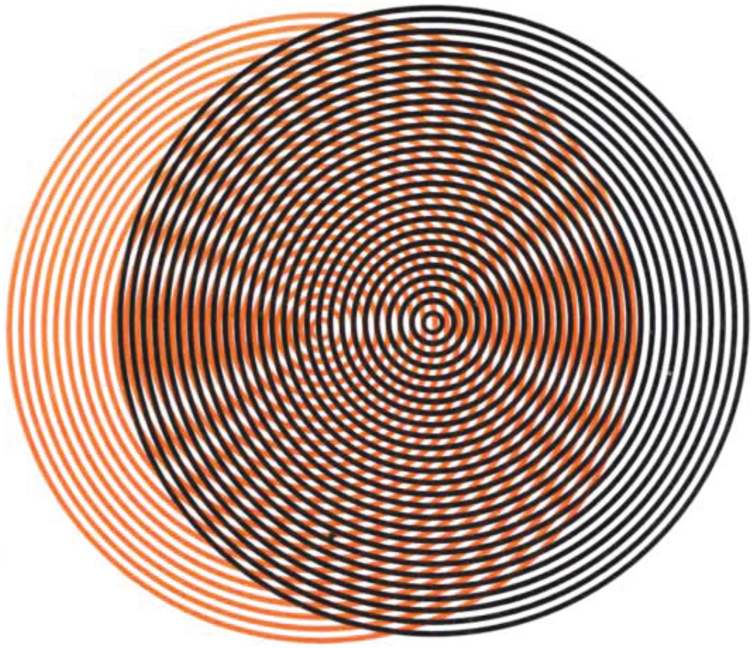




FRESNEL-RING MOIRÉS are based on the "Fresnel zone plate" (*far left*), in which the area of every ring, whether open or filled, equals the area of the center spot. When two zone plates are overlapped, the moiré consists of a series of straight lines (*second*



panels. As the two sets of overlapping circles are moved apart, the number of radiating moiré bars (actually hyperbolas) increases.

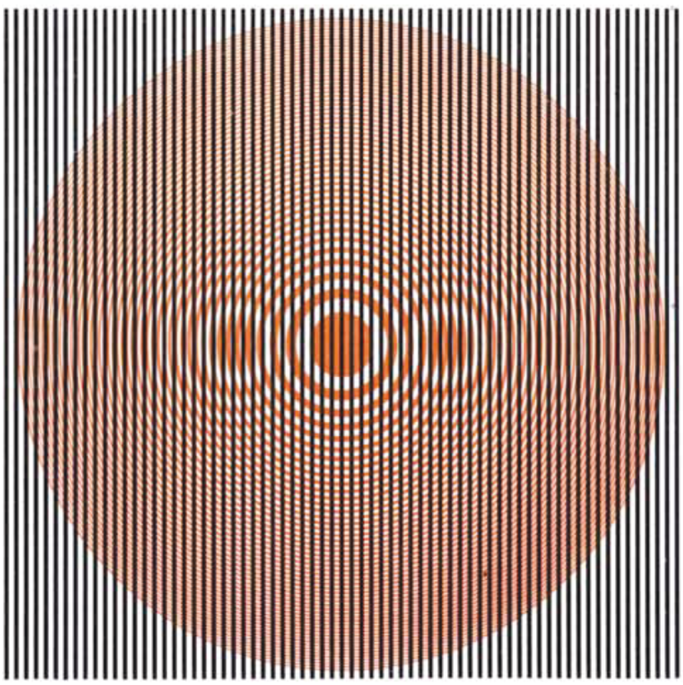


These moirés duplicate the interference patterns produced when light waves from a common source pass through two pinholes.

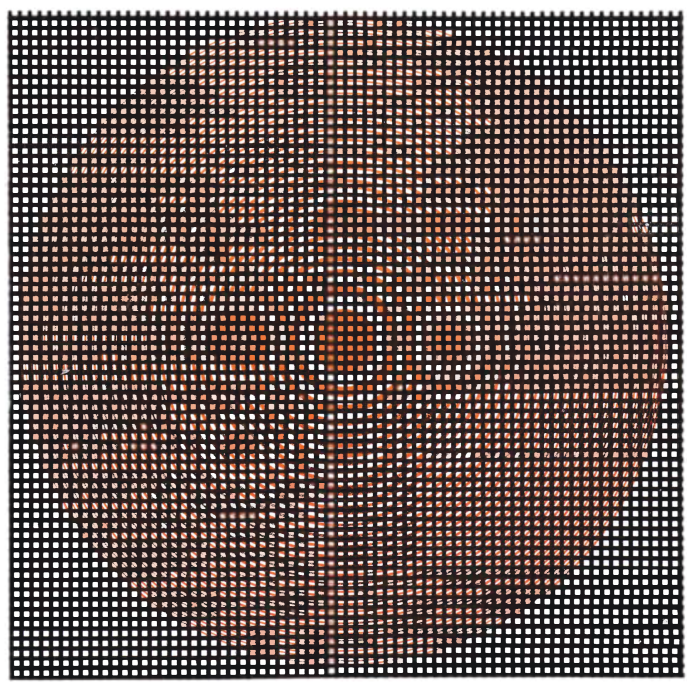
can evaluate combinations of two or more lenses. An extension of the method could be used to design lenses without the need for complicated mathematical computations.

Many novel moiré patterns can be produced by figures containing circular patterns. When two sets of concentric circles are placed together slightly off center, remarkable moiré patterns appear that change rapidly as the circles are moved. In fact, a single figure consisting of concentric circles will produce moiré afterimages in the eye and appear to revolve if the figure is moved slightly while being observed. Interacting circular patterns provide a model for many physical phenomena, such as the electrostatic fields formed by two electrically charged poles or the interference patterns produced when light from a point source passes through two adjacent pinholes in a screen. In fact, moiré patterns can be used to obtain an exact mathematical solution of the interference of light waves. They have already helped to solve problems of architectural acoustics and even to design breakwaters for harbors.

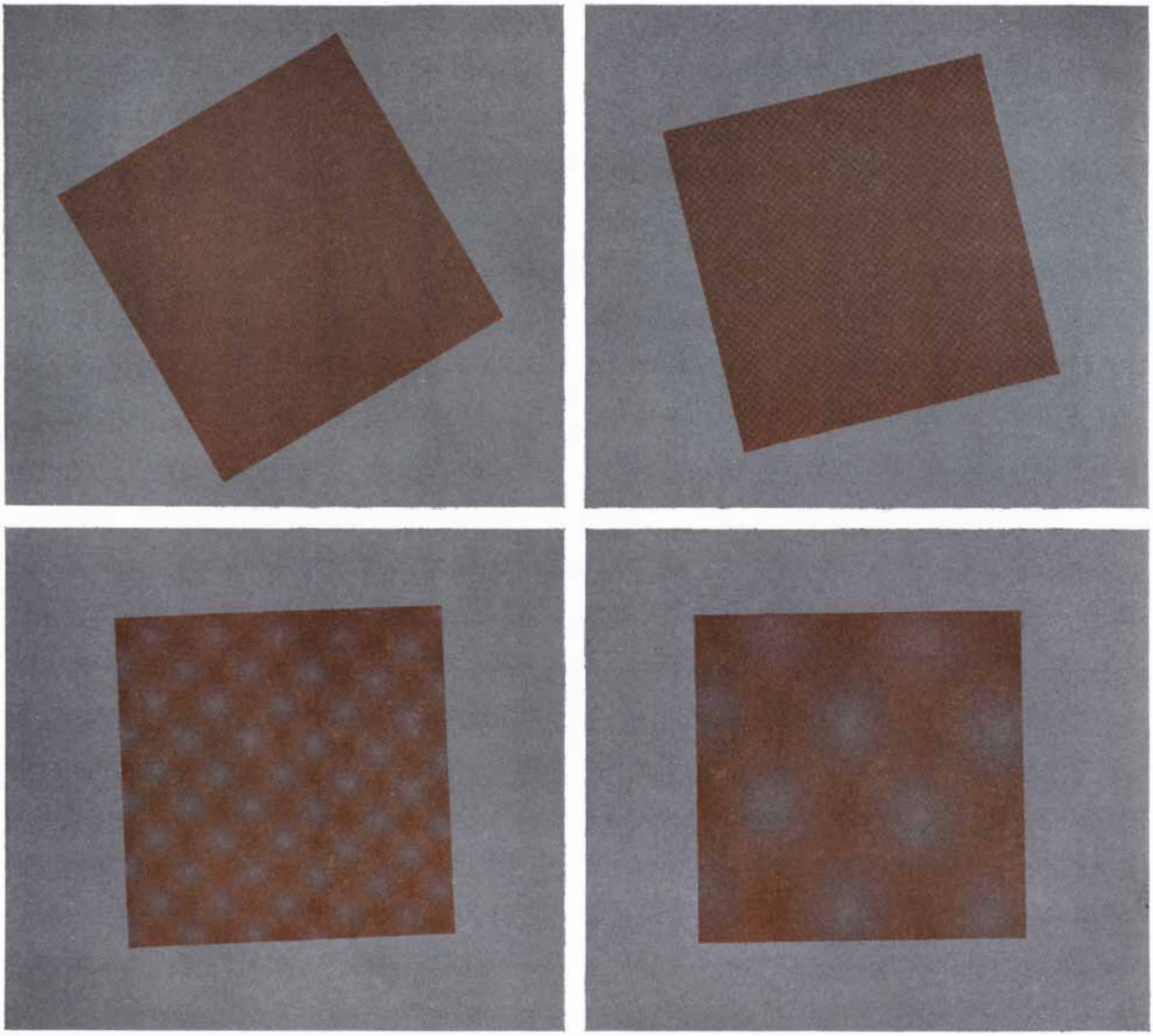
If concentric-ring patterns are drawn in a special way, the resulting moiré pattern will be a series of straight lines. To get this rather unexpected result one must use ring figures called Fresnel zone plates. The plates are made by spacing



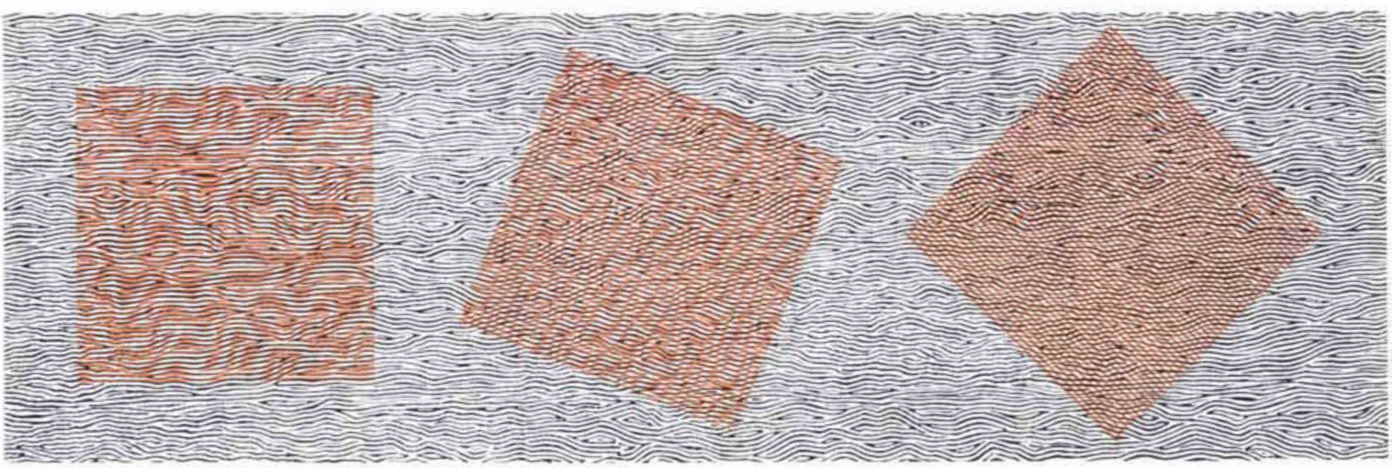
***from left*). When a grid of straight lines is placed over a zone plate, the moiré pattern replicates the zone plate (*third from left*). And**



when a second series of straight lines is placed at right angles to the first, the Fresnel zone plate is replicated again (*fourth*).



MOIRÉ MAGNIFICATION can be simply demonstrated with the aid of "halftone" screens used by engravers. The photographs in this magazine are reproduced with a screen containing 110 dots to the inch. If two such screens are to be printed one atop the other without producing a moiré, they must intersect at a fairly large angle (*top left*). If the angle is reduced, a moiré of small dots appears (*top right*). As the angle is reduced further, the dot pattern moiré is increasingly magnified, as shown in the two bottom figures.



ANALYSIS OF DISORDERLY PATTERNS can be carried out with the aid of the moiré technique. If the pattern is not wholly random, such as that in the background above, a certain amount of order will appear when the pattern is made to interact with a set of parallel lines rotated at various angles. The background pattern suggests the arrangement of long-chain molecules in a plastic.

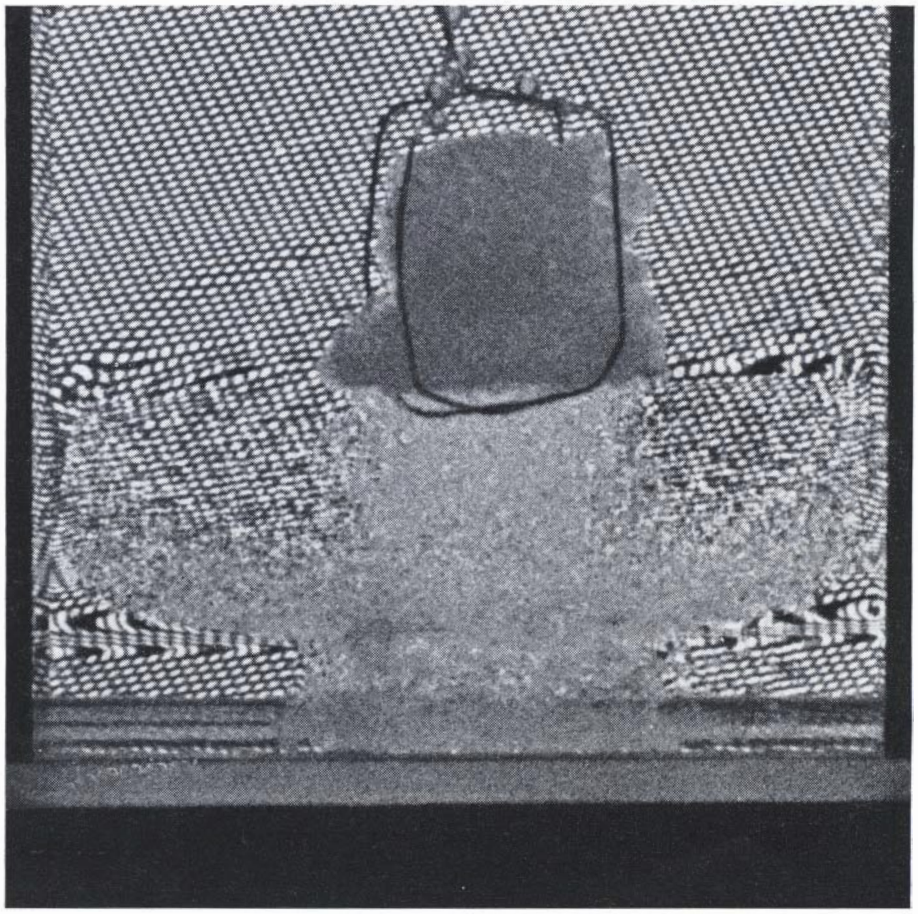
concentric circles so that the areas between them are constant; alternate areas are then blacked out. When two such plates are superposed, the moiré pattern of straight lines materializes [*see right-hand illustration at bottom of page 60*]. If a plate consisting of straight lines is placed over one Fresnel zone plate, the result is a moiré pattern composed of a series of zone plates.

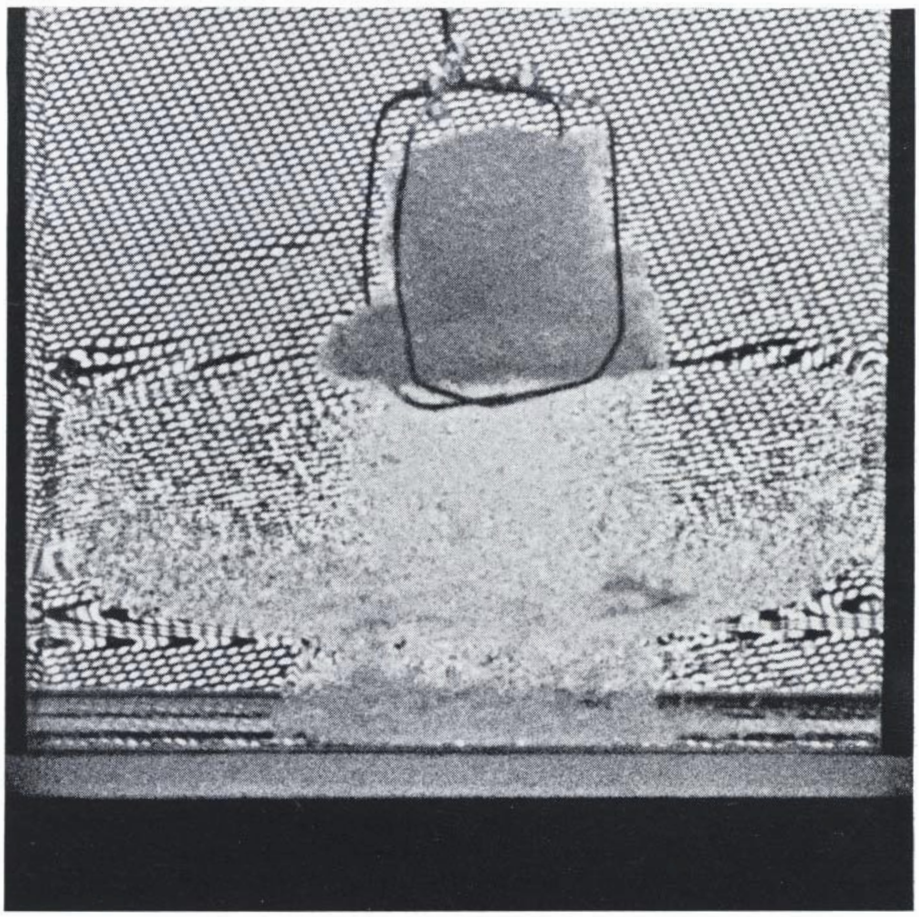
Moiré patterns can be readily created with two screens consisting of a regular pattern of fine dots. Such screens, having anywhere from 50 to 150 dots per inch, are used by photoengravers to make "halftone" reproductions and are called halftone screens. It is not surprising that moiré patterns sometimes plague the printer whenever he is obliged to print two or more halftone impressions one atop the other, which he must do in making multicolored reproductions. To avoid moiré patterns the engraver's plates must be carefully positioned so that the dot patterns intersect at about 30 degrees. One can vividly demonstrate the ability of the moiré technique to magnify an underlying pattern by reducing the angle of intersection between two halftone screens, as shown in the illustrations at the top of the opposite page.

If the underlying screen is distorted slightly, the distortion will be greatly magnified. One can use this effect to advantage as a way to study the strain induced in some object placed under stress. The halftone screen could be printed directly on the object; when viewed through a second screen, any distortion would be instantly apparent.

Another simple use of the moiré technique is for direct observation of a surface containing a periodic pattern. For example, if a parallel ruled grating is placed over a woven fabric, the character of the weave can easily be discerned. If the pattern is a complex one with little evident periodicity, the moiré technique will readily sort out the areas of regularity [*see bottom illustration on opposite page*]. In this sense the moiré method is a means of establishing correlations among statistical data.

We hope this article has given the reader some feeling for the fascinating patterns that can be created by the juxtaposition of inherently simple figures. However, the patterns themselves—the moirés—can represent the solutions of extremely complicated mathematical problems. It is because moirés provide simple analogues of complex phenomena that they have such a wide range of potential usefulness.





DIFFUSION RATES can be plotted by observing changes in moiré patterns caused by changes in the refractive index of a solution. These pictures show successive changes in moiré as a lump of sugar dissolves in a liquid contained between a pair of ruled plates.